Eur. Phys. J. C 17, 455–459 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000484

THE EUROPEAN
PHYSICAL JOURNAL C
© Società Italiana di Fisica
Springer-Verlag 2000

Bosons doubling

H. Mohrbach[1], A. Bérard[1], P. Gosselin[2]

[1] L.P.L.I. Institut de Physique, 1 blvd D.Arago, 57070 Metz, France
[2] Université Grenoble I, Institut Fourier, UMR 5582 CNRS-UJF, UFR de Mathématiques, BP74, 38402 Saint Martin d'Hères, Cedex, France

Received: 26 June 2000 / Published online: 31 August 2000 – © Springer-Verlag 2000

Abstract. It is shown that next-nearest-neighbor interactions may lead to unusual paramagnetic or ferromagnetic phases which physical content is radically different from the standard phases. Actually there are several particles described by the same quantum field in a manner similar to the species doubling of the lattice fermions. The renormalizability of the theory is proven at the one loop level.

1 Introduction

The most practical way to build a physically relevant quantum field theory starts with a choice of a suitable Lagrangian. More than the Hamiltonian formalism, the Lagrangian enforces both the Lorentz invariance and the symmetry principles. Yet the difficulty is the choice of the terms to include in the Lagrangian allowed by the symmetries. This choice has been dictated for a long time by the principle of renormalizability. Solely renormalizable quantum field models were considered as sensible physical theories in particle physics. The need of infinitely many coupling constants to cancel the UV divergences generated in the framework of the perturbation expansion made the non-renormalizable theories be rejected. Today any realistic quantum field is considered as an effective theory valuable only in a given range of energy. In this context renormalizability is no more considered as a fundamental physical requirement. It may be possible that the quantum field theories we are familiar with, are low energy approximations of a theory that may not even be a field theory.

Any effective theory includes both renormalizable and non-renormalizable interactions. But the characterization of the second ones was modified by looking into their importance at low energy. There, we expect they are highly suppressed. So non-renormalizable interactions may be excluded from the start because their influence on the dynamics decreases with the physical energy scale; i.e., they do not change the universality class of the model, but their influences grow when we consider the high energy dynamics. These non-renormalizable interactions are then interpreted as the influence of some degrees of freedom relevant at higher energy on the low energy physics. For instance, the heavy particle perturbative elimination leads to an effective Lagrangian for the light particles containing an infinite number of non-renormalizable interactions expressed in terms of the light degrees of freedom.

Consider as an example a single scalar component Lagrangian with higher derivative interactions of the form $\varphi\Box^n\varphi$. We follow the discussion of Steven Weinberg's book [1]. Such a term make a contribution of the form $(q^2)^n$ to the free propagator. Thus it would not have the simple pole expected but n such poles usually at complex value of q^2 [2]. They could be interpreted as particles with negative norm which violate unitarity [3]. Following Weinberg's argument if this non-renormalizable operator has a coefficient of order $M^{-2(n-1)}$ ($M \gg m$), then the extra poles are at q^2 of order M^2 and we can not neglect all the other non-renormalizable interactions. In other words the higher derivative terms are non-renormalizable interactions generated by the elimination of a particle of mass M. It is then difficult to describe the physics beyond the heavy particle threshold without this particle as a dynamical degree of freedom. To due this job without the true degrees of freedom we precisely need the fine tuning of an infinity of non-renormalizable interactions. Therefore the truncation of the effective Lagrangian to terms of the form $\varphi\Box^n\varphi$ is a poor approximation.

In this letter we consider a one component scalar field theory with higher derivative interactions regularized on a lattice. Usually, a lattice or a continuum theory differ only by non-renormalizable interactions leading to the same low energy physics. But it is well known that in some cases, singular configurations or topological defects at the scale of the cutoff may appear on the lattice, polluting the numerical simulation. Following the previous discussion the usual attitude is to suppress such configurations by improving the action [4] in order to recover the same continuum limit than in renormalized perturbation theory. In the present letter we consider a model with a next-nearest-neighbor interaction and choose to vary the dynamics close to the cutoff scale to look if this may change the physics at large distance. Then at first sight, it seems that the preceding reasoning would forbid such a point of vue: the true degrees of freedom or all non-renormalizable interactions are needed at the scale of the cutoff. This is true if we our theory is an effective low energy theory. But if it appears to be renormalizable, our model is correct at all energy scales and we don't need to include all other non-renormalizable interactions in the action.

Keep in mind that the classification of the interactions is usually done by simple dimensional analysis (power counting arguments in perturbation theory) but that non-perturbative study may invalid this classification. For example, the formation of small positronium bound states (of the size of the cutoff) in strong massless QED, generates by taking into account the anomalous dimensions, new relevant (renormalizable) operators. The condensate of these bound states breaks the chiral symmetry. The IR feature of the resulting vacuum are thus modified compared to the perturbative one [5], [6].

Our goal is to look if continuum physics exists beyond the class of traditionally renormalizable theories. By considering our theory on a lattice we find a free propagator containing many minima (in euclidean space). These ones are similar to the doubling fermions and will be interpreted as different particles, some of them having different masses. By a particular fine tuning of the coupling constants, we find a renormalizable theory at least at the one loop level. We also introduce a 16-components field Φ_α, in such a manner that each component is responsible of the excitations around one minima. In this manner each of these excitations is now interpreted as low energy excitations of different fields. This formalism allows us compute the one loop effective potential.

The present letter is a generalization of a preceding work published in two papers [7] where the accent was put on the breakdown of the Poincaré symmetry by the second pole of a propagator containing two minima, leading to an antiferromagnetic vacuum. The lattice is a good regulator since contrary to the other ones it regularizes the quantum fluctuations as well as the saddle point. This may be important if non-homogeneous saddle point are present. In the present paper we will work with a trivial vacuum, the generalization to a ferromagnetic one being trivial.

2 The model

We consider the following single component scalar field action in a d dimensional lattice:

$$S[\varphi(x)] = \sum_x \Bigg\{ -\frac{1}{2}\varphi(x)\Big[A\varphi(x) + \sum_\mu \left(J\varphi(x+e_\mu) + K\varphi(x+2e_\mu)\right)\Big]\Bigg\} + \sum_x \left(\frac{\widetilde{m}^2}{2}\varphi(x)^2 + \frac{\lambda}{4!}\varphi(x)^4\right) \quad (1)$$

where the coefficients A, J, K are chosen to be positive. The theory describes a paramagnetic (P) or a ferromagnetic (F) phase. A negative sign for J leads to an antiferromagnetic phase with the breaking of the Lorentz invariance. We don't want to discuss such a situation here. It is well known from renormalization group argument that next-nearest-neighbor ferromagnetic coupling are irrelevant for the description of the P or F phase at least near the phase transition. In particle physics language those operators have a decreasing influence on the dynamics as we move away from the UV scaling regime towards the physical energy scales, in other words they do not change the universality class of the model. In a P or F phase the important modes are the modes near zero. In particular they are responsible for the instability leading to a phase transition. It will be shown below that for this model all the relevant modes lie in fact around each edge of the Brillouin zone. These fast fluctuating modes are then relevant as precursor of a phase transition to an antiferromagnetic phase. We aim to study the influence of these modes in the continuum limit. As usual, the fluctuations around a minimum of the propagator are interpreted as particle like excitations. So we will show that our model describes the dynamics of 2^d particles. This is similar to the fermion doubling on the lattice except that our particles are not degenerates.

3 The elementary excitations

The particles in the mean-field approximation are given by the free propagator:

$$G^{-1}(p) = -A + \widetilde{m}^2 - 2\left(J\sum_\mu \cos p_\mu + K\sum_\mu \cos 2p_\mu\right) \quad (2)$$

which has the particularity to have 2^d minima in each edge of the Brillouin zone if:

$$K > \frac{J}{d} \quad (3)$$

It's advantageous to divide the Brillouin zone

$$\mathcal{B} = \{p_\mu, |p_\mu \leqslant \pi|\}, \quad (4)$$

into 2^d restricted zones,

$$\mathcal{B}_\alpha = \left\{|p_\mu - P_\mu(\alpha)| \leqslant \frac{\pi}{2}\right\} \quad (5)$$

whose centers are at

$$P_\mu(\alpha) = \pi n_\mu(\alpha), \quad (6)$$

where $n_\mu(\alpha) = 0, 1$ and the index $1 \leqslant \alpha \leqslant 2^d$ is given by

$$\alpha = 1 + \sum_{\mu=1}^{d} n_\mu(\alpha)\, 2^{\mu-1}. \quad (7)$$

The propagator for the zone $\mathcal{B}_\alpha$ is $G_\alpha^{-1}(q) = G^{-1}(P(\alpha) + q)$. It turns out that all the Brillouin zones $\alpha = 1..2^d$, contain particle like excitations.

In particular:

$$G_1^{-1}(0) = -A - 2d(J+K) + \widetilde{m}^2 \quad (8)$$

So we chose arbitrary: $A = -2d(J+K)$. If we assume that the mass term is finite, the free propagator in the limit $a \to 0$ is (with the lattice spacing explicitly reintroduced):

$$G_\alpha^{-1}(p) = Z(\alpha)p^2 + m^2(\alpha) + O(a^2p^4) \quad (9)$$

with the mass given by:

$$m^2(\alpha) = m^2 + \frac{Jd}{a^2}\sum_{\mu=1}^{d} n_\mu(\alpha) \quad (10)$$

where $m^2 = \frac{\widetilde{m}^2}{a^2}$. The condition of finiteness of the mass terms $m^2(\alpha)$ leads to a non-usual renormalization of the coupling constant J. That is, we must choose $J = \mu^2 a^2$ where μ^2 has the dimension of a mass and is kept finite. Otherwise all the particles except the one in the first Brillouin zone will decouple. In this case we recover the usual phase where only the modes around zero are relevant. It is trivial to see that the classical continuum limit is the superposition of two uncoupled sub-lattices because $J = 0$ in the limit $a \to 0$. This tree level renormalization $J = \mu^2 a^2$ may appear unusual. However remember that only the physical masses and coupling constants have to be cut-off independent, not the bare parameters.

We also choose $K = \frac{1}{d}$ to get $Z(\alpha) = 1$ in the continuum limit. The appearance of the new minima is precursor of an antiferromagnetic instabilities. That is, configurations with some antiferromagnetic directions are metastable states in the paramagnetic phase.

With this choice for the couplings we will prove at the one loop order that our model describes a well defined renormalizable field theory with 2^d interacting particles.

4 The perturbation expansion

We follow the standard procedure by computing the different 1-PI function at the one loop level in $d = 4$. As the initial action has only one field, it is not trivial that the UV divergencies may be cancelled only by one mass and one coupling counter term (it's easy to check the wave function renormalization constant is $\delta Z = 0$ at the one loop order).

We start with:

$$\begin{aligned} \Gamma^2(k,-k) &= G^{-1}(k) + \frac{g}{2}\int_{\mathcal{B}} G(p) \\ &= G_\alpha^{-1}(\widetilde{k}) + \frac{g}{2}\sum_\alpha \int_{\mathcal{B}_1} G_\alpha(\widetilde{p}) \end{aligned} \quad (11)$$

with $k = \widetilde{k} + P(\alpha)$, and $\widetilde{k} \in \mathcal{B}_1$. As usual we replace the bare coupling by the renormalized one. The physical renormalized mass is:

$$m^2 = \Gamma^2(0) = m^2 + \delta m^2 + \frac{g}{2}\sum_\alpha \int_{\mathcal{B}_1} G_\alpha(\widetilde{p}) \quad (12)$$

which defines the mass counter term, and make the two point 1-PI function finite. With this choice the other physical masses of the different particles are define unambiguously by:

$$m^2(\alpha) = \Gamma^2(P(\alpha)) = m^2 + \mu^2 d \sum_{\mu=1}^{d} n_\mu(\alpha) \quad (13)$$

The renormalization of the coupling constant is more involved. Consider:

$$\begin{aligned} &\Gamma^4(k_1,k_2,k_3,k_4) \\ &= g + \delta g - \frac{g^2}{2}\int_{\mathcal{B}} G(p)G(k_1+k_2+p) + \text{Perm} \quad (14) \\ &= g + \delta g - \frac{g^2}{2}\sum_\alpha \int_{\mathcal{B}_1} G_\alpha(\widetilde{p})G_\alpha(k_1+k_2+\widetilde{p}) + \text{Perm} \end{aligned}$$

The renormalized coupling constant is defined as usual as $g = \lim_{a\to 0} \Gamma^4(0)$ so that the counter term is:

$$\begin{aligned} \delta g &= \frac{3g^2}{2}\sum_\alpha \lim_{a\to 0}\int_{\mathcal{B}_1} G_\alpha(\widetilde{p})^2 \\ &= \frac{3g^2}{2}\left(\sum_\alpha \left(\frac{1}{16\pi^2}\ln\frac{\Lambda^2}{m^2(\alpha)} - 1\right) + F\right) \end{aligned} \quad (15)$$

where F is a finite part due to the lattice structure, independent of $m^2(\alpha)$ (for a detailed analysis of such integrals, see [7]). It's clear that when all the external momenta belong to $P(16)$, we have $\Gamma^4(P(16)) = \Gamma^4(0) = g$. In the other cases, it is not yet clear that the unique counter term (15) remove the UV divergencies. In fact for the renormalization of the other coupling constant we have to compute integrals of the following form where $\overline{\alpha}$ is an implicit function of α:

$$\begin{aligned} &\sum_\alpha \lim_{a\to 0}\int_{\mathcal{B}_1} G_\alpha(\widetilde{p})G_{\overline{\alpha}}(\widetilde{p}) \\ &= \sum_\alpha \int_{\mathcal{B}_1} \frac{dp}{(p^2+m^2(\alpha))(p^2+m^2(\overline{\alpha}))} + F \\ &= \sum_\alpha \frac{1}{16\pi^2}\left(\ln\frac{\Lambda^2}{m^2(\alpha)} - \frac{m^2(\overline{\alpha})}{m^2(\alpha)-m^2(\overline{\alpha})} \right. \\ &\quad \left. \times \ln\frac{m^2(\alpha)}{m^2(\overline{\alpha})}\right) + F \end{aligned} \quad (16)$$

where the finite term F is the same as above [7]. Then it is clear that for every external momenta at the edge of the Brillouin zone, the choice of the counter term (15) will remove the UV divergencies. As an example consider the following vertex function:

$$\begin{aligned} &\Gamma^4(P(16),P(16),0,0) \\ &= g + \delta g - \frac{2g^2}{2}\int_{\mathcal{B}} G(p)G(P(16)+p) \\ &\quad - \frac{g^2}{2}\int_{\mathcal{B}} G(p)G(-p) \end{aligned}$$

$$= g + \delta g - \frac{2g^2}{2} \sum_\alpha \int_{\mathcal{B}_1} G_\alpha(\widetilde{p}) G_{17-\alpha}(\widetilde{p}) - \frac{g^2}{2} \sum_\alpha \int_{\mathcal{B}_1} G_\alpha(\widetilde{p})^2 \tag{17}$$

Or

$$\sum_\alpha \lim_{a\to 0} \int_{\mathcal{B}_1} G_\alpha(\widetilde{p}) G_{17-\alpha}(\widetilde{p}) = \sum_\alpha \lim_{a\to 0} \int_{\mathcal{B}_1} G_\alpha(\widetilde{p})^2 - \sum_\alpha \frac{1}{16\pi^2} \frac{m^2(17-\alpha)}{m^2(\alpha) - m^2(17-\alpha)} \times \ln \frac{m^2(\alpha)}{m^2(17-\alpha)} \tag{18}$$

and finally:

$$\lim_{a\to 0} \Gamma^4(P(16), P(16), 0, 0) = g + \delta g - \frac{3g^2}{2} \sum_\alpha \lim_{a\to 0} \int_{\mathcal{B}_1} G_\alpha(\widetilde{p})^2 - \sum_\alpha \frac{1}{16\pi^2} \frac{m^2(17-\alpha)}{m^2(\alpha) - m^2(17-\alpha)} \ln \frac{m^2(\alpha)}{m^2(17-\alpha)} \tag{19}$$

which with the help of (15) defines another renormalized coupling constant.

5 The Beta function

We deduce the following beta function from the choice of the counter term of the coupling constant:

$$\beta(g) = m \frac{\partial g}{\partial m}\bigg|_{\lambda_0, \Lambda} = 6g^2 \sum_\alpha \frac{m^2}{m^2(\alpha)} \tag{20}$$

which gives the flow for the coupling constant:

$$g(\Lambda) = \frac{\lambda(\Lambda_0)}{1 - \frac{3\lambda}{16\pi^2} \sum_\alpha (\frac{m^2}{m^2(\alpha)}) \ln \frac{\Lambda}{\Lambda_0}} \tag{21}$$

Remark that the sign of the beta function may be negative if $\mu^2 < 0$. But a careful look shows that this happens only when $m^2 < -16\mu^2$ where the trivial vacuum is instable against the antiferromagnetic one. So we must start again the computation by considering fluctuations around this antiferromagnetic vacuum. In this case we found the same beta function. It is then clear that our theory will be trivial, or in other words the coupling constant is a nonrenormalizable one, as for each scalar theory in $d = 4$.

6 Effective potential

To deduce the universality class of the model the simplest way is to introduce a formalism allowing us to compute the one loop effective potential. It is defined as the generator function for the 1PI function as:

$$V_{eff}(\Phi) = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{\alpha_1,...,\alpha_n} \Phi_{\alpha_1}...\Phi_{\alpha_n} \Gamma^{(n)}(P(\alpha_1), ..., P(\alpha_n)) \tag{22}$$

where we have introduced a 16-components field Φ_α, in such a manner that the α-th component will be responsible of the excitations in B_α. In this manner each of these excitations is now interpreted as low energy excitations of different fields. Thus the Feynman rules are those of a 16-component field with the matrix propagator G where $G_{\alpha,\beta}(p) = \delta_{\alpha,\beta} G(P(\alpha) + p)$, and each external line with $p = 0$ is represented by the insertion of the matrix:

$$\overline{\Phi} = \sum_{\alpha=1}^{2^d} \gamma^\alpha \Phi_\alpha \tag{23}$$

where:

$$\gamma^\alpha_{\rho,\sigma} = \prod_{\mu=1}^{d} \delta_{\sigma_\mu + \alpha_\mu - \rho_\mu (\mathrm{mod}\ 2)\ ,\ 0} \tag{24}$$

takes care of the change of particle type at each vertex. Then for example at one loop:

$$\frac{1}{4!} \sum_{\alpha_1,...,\alpha_4} \Phi_{\alpha_1}...\Phi_{\alpha_4} \Gamma^{(4)}(P(\alpha_1), ..., P(\alpha_4) = \frac{g^2}{4} \sum_{\alpha_1,...,\alpha_4} \Phi_{\alpha_1}...\Phi_{\alpha_4} \int_{\mathcal{B}_1} dp Tr[G(p)\gamma^{\alpha_1} \times G(p)\gamma^{\alpha_2} G(p)\gamma^{\alpha_3} G(p)\gamma^{\alpha_4}] = \frac{g^2}{4} \sum_{\alpha_1,...,\alpha_4} \int_{\mathcal{B}_1} dp Tr[G(p)\overline{\Phi} G(p)\overline{\Phi} G(p)\overline{\Phi} G(p)\overline{\Phi}] \tag{25}$$

Taking advantage of the matrix formalism introduced above we obtain:

$$V^{(1)}_{eff}(\Phi) = \frac{1}{2} \int_{p \leqslant \frac{\pi}{2a}} \frac{d^d p}{(2\pi)^d} tr \ln \left(G^{-1} + \frac{g}{2} \overline{\Phi}^2\right) \tag{26}$$

the tree level -part of the effective potential is:

$$V^{(0)}(\Phi) = \sum_\alpha (G^{-1}_\alpha + \delta m^2) \Phi^2_\alpha + \frac{g + \delta g}{4!} \times \left(\sum_\alpha \Phi^4_\alpha + 3 \sum_{\alpha,\beta} \Phi^2_\alpha \Phi^2_\beta \right) \tag{27}$$

It is now easy to check that the model defined by the action (1) lies in the same universality class as a 16 components scalar field theory defined with only one mass counter term and one coupling constant counter term whose Lagrangian is:

$$L = \frac{1}{2} \sum_\alpha \partial_\mu \varphi_\alpha \partial^\mu \varphi_\alpha + \sum_\alpha \frac{m^2_\alpha + \delta m^2}{2} \varphi^2_\alpha + \frac{g + \delta g}{4!} \times \left(\sum_\alpha \varphi^4_\alpha + 3 \sum_{\alpha,\beta} \varphi^2_\alpha \varphi^2_\beta \right) \tag{28}$$

Nevertheless an important difference is that the masses of the particles in the model (1) are related to each other and are then not arbitrary as in the model (28).

Note that it is possible to remove most of the modes in the continuum limit by adding a diagonal interaction. In this case the theory describes a low energy dynamic of the two components scalar theory [7].

The analysis of the present paper may be applied to other models too. For example consider the following variant of the XY model defined by the action:

$$S = \frac{1}{2T} \sum_i \sum_\mu \{\alpha \cos(\theta_i - \theta_{i+\mu}) + \gamma \cos(\theta_i - \theta_{i+2\mu})\}$$

This theory will describe the usual excitations, that is the spin waves, the vortex as well as the doubling of the modes (similar to rotons excitations). It would be interesting to study the influences of theses modes on the phase transitions.

7 Conclusion

We have studied a one component scalar field theory in a d dimensional lattice with next-nearest-neighbor interaction at the one loop level. One can identify 2^d particle like excitations, and then eliminate the one-loop divergencies by an appropriate fine tuning of the bare parameters. One should emphasize that the renormalized continuum theory exists only when the regulator is taken into account both at the tree (through the free propagator) and the one loop levels in a systematical manner. The resulting theory is, equivalent to an usual renormalizable 2^d scalar field theory at low energy, but to be really conclusive the proof must be extended to higher orders in the loop expansion. It is also interesting to pursue this work by adding competing interactions in order to study if they may lead to continuum physics beyond the class of traditionally (perturbative) renormalizable theory.

Acknowledgements. H.M. thanks V. Branchina and J. Polonyi for useful discussions.

References

1. S. Weinberg, The Quantum Theory of Fields, volume I, Cambridge University Press, 1995
2. C. Liu, K. Jansen and J. Kuti, Nucl. Phys. B34, 635, 1994; Nucl. Phys. B42, 630, 1995
3. T.D Lee, G.C. Wick, Nucl. Phys. B9, 209, 1969; Phys. Rev. D2, 1033, 1970
4. P. Hasenfratz and F. Niedermayer, Nucl. Phys. B414, 785, 1994
5. W.A. Barden C.N. Leung and S.T. Love, Nucl. Phys. B323, 493, 1989; Nucl. Phys. B273, 649, 1986
6. P.I. Fomin and al, Riv. Nuovo Cimento 6, 5, 1983
7. V. Branchina J. Polonyi, H. Mohrbach, Phys. Rev. D60, 45006, 1999; Phys. Rev. D60, 45007, 1999